J. Fluid Mech. (1968), *vol.* 34, *part* 1, *pp.* 159–162
Printed in Great Britain

Determination of the direction of surface velocity gradients in three-dimensional boundary layers

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(Received 12 February 1968)

An instrumental method is proposed to measure the direction of surface velocity gradients in three-dimensional boundary layers. It utilizes electrochemical techniques developed in this laboratory.

1. Introduction

In three-dimensional boundary layers the direction of the viscous force on the surface can be different from the direction of the external flow. Skin friction lines can be defined which are tangent to the vectors describing these surface forces. As has been pointed out by Lighthill (Rosenhead 1963), the determination of the pattern of these skin friction lines is needed in order to understand three-dimensional boundary layers. This pattern is usually approximated by visual experiments. The advantages and disadvantages of these visual techniques have been described by Pankhurst and Gregory (Rosenhead 1963).

This paper describes how the direction of surface velocity gradients (or of surface forces, if the viscosity is known) can be measured by employing an instrumental rather than a visual technique. The new instrument utilizes electrochemical techniques which have been used in this laboratory to study turbulence close to a wall (Reiss & Hanratty 1963; Mitchell & Hanratty 1966; Hanratty 1967) and to study two-dimensional boundary layers (Dimopoulos & Hanratty 1968; Son 1968).

2. Theory

The electrochemical reaction described by Mitchell & Hanratty (1966) is carried out on the surface of a platinum electrode embedded in a solid surface at high enough voltages that the current flowing in the electrochemical circuit, I, is controlled by the rate of mass transfer to the surface and the concentration of the reacting species at the wall is zero. A mass transfer coefficient, $\langle K \rangle$, can be calculated as

$$\langle K \rangle = I/AFC_B \tag{1}$$

where A is the surface area of the test electrode, F is Faraday's constant and C_B is the bulk concentration. Consider the rectangular electrode shown in

figure 1 with dimensions (L, W) smaller than the radius of curvature, r, of the solid surface and $L \ll W$. Since the diffusion coefficient, D, of the reacting species (ferricyanide ion) and the length, L, are quite small the concentration boundary layer is contained entirely in a region where the velocity gradient is constant. The velocity gradient at the surface, S, can therefore be calculated from the measured mass transfer coefficient by the following equation derived by Mitchell (1965):

$$\langle K \rangle = \sigma \left(\frac{S \sin \beta}{L} \right)^{\frac{1}{3}} \left(1 + \frac{L}{5W} \cot \beta \right), \tag{2}$$

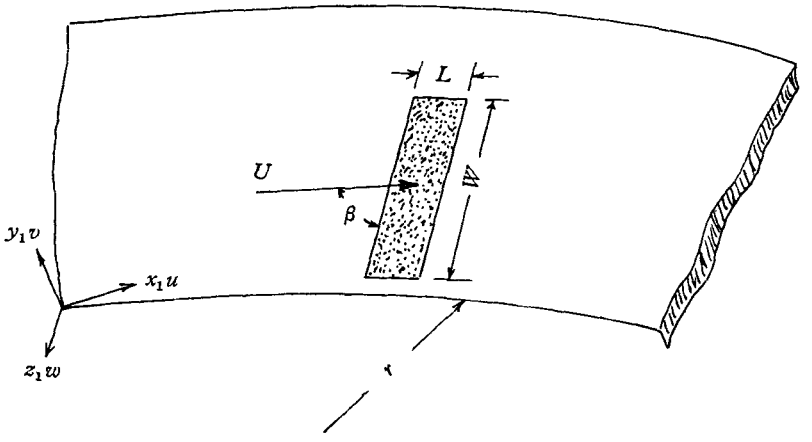

FIGURE 1. Electrode arrangement.

where β is the angle which the electrode makes with the velocity of the fluid, U, close to the wall. The coefficient σ can be calculated and need not be determined by a calibration,

$$\sigma = \left[\frac{3D^{\frac{2}{3}}}{2\Gamma(\frac{4}{3}) 9^{\frac{1}{3}}} \right]. \tag{3}$$

The term $$\sigma \left(\frac{S \sin \beta}{L} \right)^{\frac{1}{3}} \left(\frac{L}{5W} \cot \beta \right)$$

in (2) accounts for edge effects. The direction β can be determined by measuring the currents to two electrodes, 1 and 2, which are oriented perpendicular to each other. A first approximation to the angle, β_0, can be calculated from the currents to the two electrodes,

$$\beta_0 = \tan^{-1} \left\{ \left(\frac{\langle K_1 \rangle}{\langle K_2 \rangle} \right)^3 \left(\frac{L_1}{L_2} \right) \right\} = \tan^{-1} \left\{ \left(\frac{I_1 A_2}{I_2 A_1} \right)^3 \frac{L_1}{L_2} \right\}. \tag{4}$$

A second approximation to β is obtained from the following equation:

$$\left(\frac{\langle K_1 \rangle}{\langle K_2 \rangle} \right) \left(\frac{L_1}{L_2} \right)^{\frac{1}{3}} = (\tan \beta)^{\frac{1}{3}} \left(1 + \frac{L_1}{5W_1} \cot \beta_0 \right) \Big/ \left(1 + \frac{L_2}{5W_2} \tan \beta_0 \right). \tag{5}$$

3. Design of the instrument

The derivation of (2) used the assumption that the flow is uniform over the electrode surface. The design of the instrument is largely a compromise between making it small enough so the measurements will not be influenced by non-uniformities in the flow and yet large enough that edge effects are not too important. The probes shown in figure 2 were fabricated for use in a cylinder having a diameter of 3 in. Two pairs of electrodes are used in order to obtain a better average at the centre of the probe. The auxiliary electrodes 1 and 3 determine

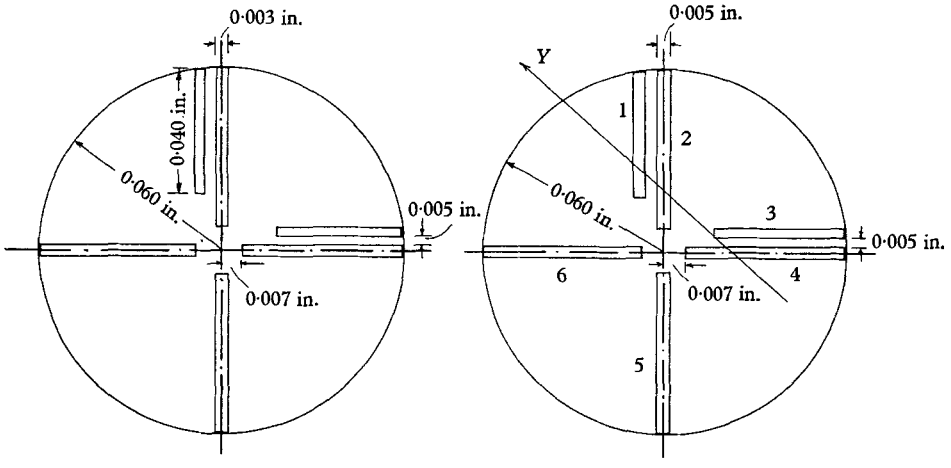

FIGURE 2. Typical dimensions of the probes.

roughly the direction of flow at the beginning of the measurements. By activating first electrode 1 and then electrode 2 we observe a reduction of the diffusion current of electrode 1 if the local velocity, U, is in the Y-direction (see figure 2). This sandwiched arrangement of electrodes has been used before (Son 1968) to determine the direction of the wall stress and the separation point for two-dimensional boundary layers.

The probes shown in figure 2 were fabricated in the following manner: cuts were made parallel to, and with the bottoms at a distance about 0·007 in. from, the main axis of a 0·250 in. Plexiglas cylinder. Platinum sheets were glued in the gaps. The cylinder was turned in a lathe to 0·120 in. and the lead wires were attached to the electrodes by electrically conductive cement. The probe was then fixed with epoxy resin in a hole in the 3 in. Plexiglas tube used in the tests. It was sanded flush to the surface of the tube and polished. An optical comparator was used to determine the orientation of the electrodes with respect to a reference line on the outside of the tube.

4. Tests

The tube was placed with its axis perpendicular to the flow direction in the test section of the gravity-fed water tunnel used by Dimopoulos & Hanratty (1968). The probes were located 45° from the front stagnation point. Four probes having

three different orientations with respect to the direction of mean flow were investigated. The angle β is now defined as the angle between the direction of the mean flow and electrode 6. Two separate electronic circuits similar to the one described by Mitchell & Hanratty (1966) were used so that the signals from a pair of electrodes could be measured simultaneously. Some fluctuations of low frequency existed in the flow so a number of values of the ratio of two signals were measured over a sufficiently long period of time.

Approximate electrode thickness (in.)	Reynolds number	Angle of orientation (β)	Electrodes 6 and 2		Electrodes 4 and 5		Average error
			Measured angle β	Error	Measured angle β	Error	
0·003	600	78·75°	75·34°	−3·41°	74·14°	−4·61°	−4·01°
0·005	600	69·50°	65·15°	−4·35°	71·88°	+2·38°	−0·98°
0·005	600	43·50°	45·05°	+1·55°	43·62°	+0·12°	+0·83°
0·003	600	43·50°	44·60°	+1·10°	49·05°	+5·55°	+3·32°
0·003	775	78·75°	74·60°	−4·15°	72·77°	−5·98°	−5·06°
0·005	775	69·50°	65·46°	−4·04°	65·45°	−4·05°	−4·04°
0·005	775	43·50°	43·71°	+0·21°	45·16°	+1·66°	+0·93°
0·003	775	43·50°	43·91°	+0·41°	47·10°	+3·60°	+2·00°

TABLE 1. Results for probes at a 45° angle from the front stagnation point

Table 1 shows some of the results obtained. The average error varies between 1 and 5 degrees. The major sources of error seem to arise from inaccuracies in the determination of the dimensions of the electrodes and of their orientation on the surface of the cylinder.

Acknowledgment is made to the donors of the Petroleum Research Fund, administered by the American Chemical Society, for support of this work.

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